"The Minds, They Are A Changing"

By

Danielle Sturtz,
Linda Margusity & Rodger Smith

Pennsylvania State University
State College Area School District
Spring, 2004

Table of Contents
"The Minds, They are a Changing"

"The Minds, They Are A Changing"
Danielle Sturtz, Linda Margusity & Rodger Smith

A Brief Introduction

Three of us have designed and implemented our inquiry: Danielle Sturtz (Dani) who is a current intern, Linda Margusity, a second grade mentor, and Rodger Smith, a Professional Development Associate (PDA). We each bring a unique perspective and voice to the project. We began with the same questions, yet over the course of the inquiry each of us has branched into other questions of interest. The first section will give the background information for each participant. Each person's section will end with his/her wonderings. The background section will conclude with the original wonderings we created together. Next we will discuss the inquiry work, which the three of us did together, and what we learned. Since the three of us ended up with a different focus, we will each share our thoughts about possible future directions.

Three Voices-Our Backgrounds

Dani's Voice

Math has been my least favorite subject since sixth grade. I attribute most of my distaste for mathematics to my math teachers and the methods they used to teach the subject. I vividly remember liking mathematics until I reached the sixth grade. My sixth grade math teacher was extremely stern and showed no mercy. She always put students on the spot. Whether my hand was raised or not, I could always count on her to choose me to answer a question. She conveniently managed to call my name when I was not confident in my answer. I recall feeling uncomfortable and uneasy the entire period for fear that she would call on me and I would not know the correct answer. I thought if I answered incorrectly all the other students would laugh at me and I would be mortified. I would avoid eye contact at all costs in hopes that she would not notice me and choose someone else to answer the question. This experience has marked the start of my negative feelings about mathematics. That is, until now.

As a PDS intern, I have been provided wonderful opportunities to explore inquiry within my classroom. This is my first experience with this type of inquiry and I must say it has been extremely interesting. I feel I have benefited from the

chance to participate in the inquiry process with my mentor teacher and my PDA. This exploration has taught me a great deal about the amount of time it actually takes to successfully complete an inquiry.

Determining what topic to choose for my inquiry was not an easy task. I was unsure of where to start so I decided to narrow my topics down to two categories; something I truly enjoyed or something I disliked. Now you may ask why I would choose to do my inquiry project on something I did not like. However, I believe deep passion can stem from aversion.

My wonderings developed from my fear of how I would be able to teach mathematics to my students when I did not like the subject and lacked confidence in myself. I wanted to figure out the best way to teach my students mathematics to ensure they would never feel the way I felt in every math class I was in ever since sixth grade. I decided to do my inquiry in collaboration with Linda Margusity and Rodger Smith, focusing on reform mathematics.

There are various views about a reform mathematics curriculum, both positive and negative. I read through *Investigations in Number, Data and Space* to see what this curriculum offered and *Elementary and Middle School Mathematics* by John Van De Walle to gain some insight about a reform mathematics curriculum. Van De Walle believes five shifts in the environment of the mathematics classroom

are necessary to allow for student growth in mathematics.

1. Toward classrooms as mathematics communities and away from classrooms as individuals.
2. Toward logic and mathematical evidence as verification and away from the teacher as the sole authority for right answers.
3. Toward mathematical reasoning and away from memorizing procedures.
4. Toward inference, inventing, and problem solving and away from an emphasis on the mechanistic finding of answers.
5. Toward connecting mathematics, its ideas, its applications and away from treating mathematics as a body of isolated concepts and procedures. (1998, p. 6)

Investigations Series is one type of reform mathematics curriculum, which provides opportunities for the student growth in mathematics that Van De Walle discusses. The beginning of each book in *Investigations* discusses the *Investigations* curriculum and how to use the book. Each book contains "Investigations", with a varying number of sessions pertaining to the main topic. In the beginning of each Investigation, the first two pages are designated to tell the teacher what happens in each session, the mathematical emphasis, and what to plan ahead of time. The sessions include the materials needed, an introduction to the

lesson, a procedure for the lesson, potential discussions, and examples of student work. Throughout the sessions there are teacher's notes and dialogue boxes that provide support for the teacher. There is an Appendix about classroom routines and vocabulary support for second-language learners. In addition, blackline masters at the back of the book include a family letter and student sheets that will be used during various sessions.

The *Investigations Series* has four main goals.

1. To offer students meaningful mathematical problems
2. To emphasize depth in mathematical thinking rather than superficial exposure to a series of fragmented topics.
3. To communicate mathematics content and pedagogy to teachers.
4. To substantially expand the pool of mathematically literate students. (1997, p. 1)

The four main goals of *Investigations* coincide with Van De Walle's beliefs that five shifts in the environment of the mathematics classroom are necessary to allow for student growth in mathematics. Therefore, the *Investigations* Series would be an appropriate model of a reform mathematics curriculum according to Van De Walle. I personally have enjoyed using *Investigations* in our classroom. The strategies and activities in the series have provided my students with the opportunity for everyone to work on the same problems, yet they do so at their

own pace. They use strategies they feel comfortable with, and understand. For example, some of the students are counting all, while others are counting on to solve the problems.

My main wondering is *how does a reform mathematics curriculum affect the attitudes and knowledge of my students*? My sub questions include:

- How does this curriculum affect my attitude toward teaching math?
- How does being a facilitator help improve my children's mathematical thinking?
- How do I negotiate the tension between "telling" them and letting them figure it out for themselves?

Linda's Voice

When I began my career as an elementary teacher, I would have never seen myself in the position of someone who teaches using reform mathematics. After all, the first year I taught second grade math, I finally learned what regrouping actually meant. I have always known how to regroup, what we used to call carrying and borrowing, but I truly did not understand why it worked until I had to help first and second graders understand it.

When I began teaching, it was suggested that we use manipulatives and place

value boards by placing the cubes for the problems in the columns, and regrouping once we reached 10 or more. It seemed that kids could do what I wanted them to do, but I knew some of them did not understand why it worked. As the first couple years passed, I became more and more frustrated with my math teaching. I started to wonder if the transition from manipulative work to problems on paper might be the difficulty, so I spent a few years working this transition.

I was in this frame of mind when I heard about a mathematics project being offered through Penn State University. The "Mathematics Teacher Development Project", or MTD, was a three-year commitment for the practicing teachers and Penn State students who applied. Even though math frustrated me, I decided to apply. The three years that followed were intense. I had to change many of my preconceived notions concerning the development of children's thinking in math. Slowly I came to realize that the way I taught place value and addition and subtraction created barriers to the kids developing understandings. As it turned out, I was very good at "training" them to do what I wanted them to do; I was not helping them understand it.

Since the third year of the MTD project I struggled with better ways to teach the understanding of place value, which seemed to be the biggest concept for the kids in second grade. I used some of the resources from MTD to find

better ways to teach some of the other bigger ideas from our curriculum as well: basic math facts, double digit adding/subtracting, time and fractions.

During the last three or four years I have been using the *Investigations in Number, Data, and Space* series as my main teaching tool. I also use parts of *Elementary and Middle School Mathematics* by John A. Van de Walle as a resource. My focus has been on helping children develop their own understandings of place value, which involves applying strategies to problems. I am definitely seeing improvement in my children's ability to <u>understand</u> what place value is. Their willingness to tackle all sorts of problems, and for the most part, stick with tough problems has amazed me! My "gut feeling" is that this way of teaching mathematics is better than what I have done in the past. Yet I wondered if I was being thoughtful in my teaching, really considering the kids and their needs, or if I was using these resources to "teach without thinking".

I have learned that inquiry is based upon the idea of gaining insight into one's teaching, of systematically looking at what we choose to do. Since Rodger Smith, the professional development associate (PDA) assigned to my intern, had also participated in the MTD project, I knew that the two of us share some similar ideas about the teaching of mathematics. My hope was that Rodger would bring a different perspective, one from a third grade teacher who uses reform

mathematics and a PDA who co-taught the interns' mathematics course this fall. Dani Sturtz, my intern, was invited to participate with us. I felt she would bring a perspective of one who has not used *Investigations,* and would most likely be able to look at it from a fresh point of view. Therefore our inquiry this spring has focused on the development of children's mathematical understandings.

As part of the inquiry, I wanted to take the time to examine a few resources to determine if others support the way I teach. I am very familiar with the three books of the second grade *Investigations* series, which deal with place value, adding and subtracting. I have a good grasp of the reasoning behind the activities and the activities themselves. Since very few teachers in our district seem to use *Investigations*, I decided to start with a resource which many teachers would find familiar, *Developing Number Concepts* by Kathy Richardson. Many have used the original "brown math book", published in 1984, since it was first purchased in our district. I have had a copy for years, yet each time I read it, I did not feel it fully answered the question of how to teach children to understand, and not just do, mathematics. In 1999 Richardson published a revised set of Developing *Number Concepts*, now consisting of three books.

This inquiry is giving me the motivation to read through the parts of Richardson's books I have not explored yet. I have been focusing on the

"information" sections: explanations, goals, analyzing & assessing children's needs, questions to guide observations, and meeting the range of needs. These are the parts that describe why it is important to teach a particular way. I have looked at some of the accompanying activities as well.

With all the background I have in reform mathematics, one might think there are no wonderings left to explore. Luckily, there are always wonderings for teachers who choose to question their own teaching! Some of my wonderings around mathematics deal with furthering my understanding of teaching through reform mathematics.

- Am I being thoughtful with my teaching through *Investigations?* Am I really looking at what the kids understand?
- What do other resources say about teaching children to understand, rather than do, mathematics? Do they support the way I've been teaching mathematics?
- Are the chosen activities having an effect on the students' attitudes toward math?

Rodger's Voice

Dr. Martin Simon stood in front of our math class and made the suggestion that children should invent algorithms that made sense to them. I was shocked.

What could he be thinking? Why in the world would you trust the children to invent arithmetic? I sadly shook my head and thought that the world was being turned upside down and that would be the last math course I would take from the ivory towers on the hill at Penn State University.

Subsequent years and course work with Dr. Simon led me to question everything I felt I knew about teaching math to children. Before that, I had prided myself with my ability to teach children tricks. I got them to complete their assignments and I began many math lectures with, "You don't have to think about it. Just watch what I do."

For me, math was exact. Math had a system and logic that had to be followed relentlessly with the algorithm of the day. There were rules you had to follow and exact answers that were the "truth". I would like to publicly apologize to my students of those years. I am sorry.

I denied them the opportunity to think for themselves. Instead of building autonomy, I expected subservience. Instead of teaching them how to learn, I taught them to depend on tricks and ways of manipulating numbers that made little sense to them but "covered" the curriculum.

In part I could be excused for my behavior. I was teaching the way I was taught. I was a good math student. I got A's through high school and college and

loved the "learn a new trick each day" approach. I could do math. Or so I thought.

Through thinking, reading, writing and working on projects with other teachers, I learned that my understanding of mathematics was superficial. I could do the algorithms if you asked, but I could not explain why they worked or how you might use them in the real world.

I spent many years attending workshops and reading journal articles trying to improve my own mathematical ability. I understood for the first time that I needed to understand math in a deeper way if I were to guide my students with their learning.

I also began to realize how important it would be to understand how my third graders thought about math and how that knowledge might help them develop their own mathematical thinking. The journey was hard.

I spent many hours planning for activities that ended up being thinly disguised "do what I do" lessons. I retreated constantly from giving control to the students. I stubbornly argued with myself about the wisdom of allowing students to think for themselves. I was the teacher, right? I was the salaried employee, right? It was my job to know. It was my job to impart knowledge.

But the power of developing my own mathematical knowledge kept calling out. It was a wonderful feeling to know how things worked. You do not have to invert

and multiply when dividing fractions. You can use common denominators and divide just like you would with whole numbers. Nobody taught me that fact. I figured it out for myself in one of Dr. Simon's classes. More and more I wanted my students to experience these marvelous feelings of power and satisfaction. But I was frustrated.

I did not seem to know how to talk with them or to guide them in their own mathematical journeys. They seemed to follow strange pathways or stare blankly at me when I asked them to explain their thinking.

I came in contact with Jean Piaget's work and was even encouraged to try some of his classical tasks with the students in my room. I began to dream that I might take a sabbatical and just sit with kindergarten, first and second graders, give them problems to solve, observe them and learn how to become a better Math instructor. Family finances, however, prevented me from taking my dream sabbatical.

In the spring of 2004 I was working as a Professional Development Associate (PDA) with the Penn State/State College Professional Development School. Linda Margusity was a second grade mentor teacher at Panorama Village in the State College Area School District and I had been assigned to be the PDA for her intern, Danielle Sturtz (Dani). Linda approached me and wanted to talk about

having me participate in an inquiry project they were thinking about doing together. She wanted to do something with math and wanted to know if I would be interested in participating in some way in the inquiry.

I saw it as an opportunity to sit down with her second grade students and fulfill my dream of watching them work and grow with their mathematical abilities. It was also a chance to work with Linda who had been involved with many of the same professional development opportunities as myself. Linda and I had taken classes and course work together and I admired her enthusiasm when she talked about working with reform mathematics in her classroom. It was also an opportunity to work with a young and thoughtful student teacher who seemed interested in trying out the reform mathematics ideas that Linda was using in her classroom.

Concurrently, I had come across a book by Dr. Thomas Carpenter and his work with Cognitively Guided Instruction (CGI). Dr. Carpenter's work respects both student knowledge and teacher expertise. The following is an excerpt from the introduction of his book:

> This book is about helping you help students make sense of mathematics. My prediction is that you, too, will make sense of mathematics and more sense of how to teach. If the book is meaningful to you, you will probably have more questions than answers.

> If you are fortunate enough to teach with colleagues in this way, together you and your colleagues will begin to answer some of these questions. As your students are making sense of mathematical ideas, you will be making sense of your teaching and continuing to grow as you learn from your students, your colleagues, and your reflections. (Carpenter et al, 1999, p. x)

I could not pass up the opportunity to look at children's thinking and work with colleagues who were also interested in helping children in this way.

So Linda and Dani's invitation to participate in their inquiry project made it possible for me to explore some of the wonderings I had for years about young children's mathematical thinking.

- How do students solve problems when left on their own?
- Are there steps that all children go through when developing their problem-solving skills?
- What sort of interactions with them will help them make the transition to a more efficient way of solving problems?
- Will Cognitively Guided Instruction provide some insight into their thinking?

Our Group Wonderings

Questions

Are the activities we're doing making a difference in students' mathematical thinking?

Sub questions

- Are we being thoughtful with our teaching through *Investigations*?
- Are the *Investigations* activities carried throughout the modules at one grade level? For example, *Investigations* has many activities in two modules that really work towards kids' understandings of adding and subtracting. Are some of these activities carried through in other modules?
- Are the activities we choose having an effect on the students' attitudes toward math?
- Are the activities we choose having an effect on the adults' attitudes toward math?
- Will learning & using *CGI* allow us to be more <u>thoughtful</u> about our teaching because we will be considering what our students can do?
- Would understanding *CGI* allow us to figure out the next step for the students' mathematics, or just where the students' currently are?
- Why does changing the magnitude of the numbers seem to change the mental demand on students?

Our Inquiry Design

We began our inquiry with much discussion and found two main areas with which to start. Since both Rodger and Linda have taught using some *Investigations* activities, we decided to concentrate on creating and administering some problem types as described in *Children's Mathematics: Cognitively Guided Instruction*. (GCI) We were curious to find out if the students' experiences solving problems through *Investigations* would enable them to solve the problem types described in CGI. A total of eleven problems were created, which represented addition and subtraction as well as a few multiplication and division problems. (See Appendix A.3) When we designed the problems, we wanted to see if the kids could figure out strategies to solve them. We considered the numbers we wanted to use, and kept them below fifty so that the kids who chose counting as a strategy would not get too frustrated with counting large amounts.

We had the entire class work on them for the first day, and then used the problems as one of the centers in three subsequent days. Kids who needed more time knew to add these to their list of work to do. All kids completed most of the problems, although a few were not able to do so on their own. We then looked over their work and recorded whether they solved it correctly or not. We also recorded how every child solved each problem. From this we made a chart, which

compiled the number correct for each problem. (See Appendix B.3) For those kids who needed help to complete a problem we counted that as "incorrect", even if they did achieve a correct answer. There were a few kids who were not able to complete one of the problems, so that did not count as "correct" either.

Since we were interested to see if the kids could solve problems presented in *CGI*, we included the problem type on the chart. We wondered if there would be problem types that would be harder for the kids to solve correctly. In order to see if these were problem types they had worked on through *Investigations*, we also included a list of problem types we found in that series.

The second main area we wondered about dealt with the affective side of math. We wanted to get a sense of how our class feels about math, and how their parents perceive their child's like or dislike of math. In order to do this, we first created a parent survey, which we sent home to all families. (See Appendix A.1) Of our twenty-one families, fourteen returned the survey completed, and one additional parent sent it in with only the first question answered. (Since that survey was not complete, we did not include it in all of the data.) We assigned a numerical value to each answer, on a scale with 4 being the most desirable answer, and 1 being the least desirable answer. (See Appendix B.2) This enabled us to quantify the answers, allowing us to see a person's overall feeling about our math

program. We also could average all the answers for one particular question to see where that would fall on our scale. The last way we quantified this survey was to create an overall average.

The second survey was for the students to complete. We used faces and had them circle the face they felt best described them. (See Appendix A.2) One of us sat with a small group of students and read it to them so we could make sure they knew what each face represented. We then decided to do the same to this survey as we did to the parents' survey. We assigned points using the same scale, and used the same format to average scores. (See Appendix B.1) By using the same format for both of the surveys, we were able to use the same process to record, and later, analyze both surveys.

In the process of going over the student surveys, we realized there were some questions we wanted to ask the students. Consequently, we interviewed all twenty-one students in our classroom to provide students with the opportunity to answer in detail some questions pertaining to math. (See Appendix C) The questions included the following:

- Where did you put yourself on the survey and why?
- Do you like math and if so, what parts do you like?
- Did you do math similarly last year?
- Do you feel you are better at math this year than last year? Why?

We interviewed every student to hear their perspective on math and to see if there were any noticeable trends that occurred in their answers. After interviewing every student, Dani photocopied the notes and read through them. She then used different colored markers to highlight their answers. She made a key to distinguish between the types of questions.

What We Learned

When we began to review our data, we were surprised to see we did not learn exactly what we thought we would. However, we did discover some wonderful information from our analysis. In order to make the claims easier to follow, we organized them in three categories: reform curriculum, the affective side, and claims specific to Dani's work in the inquiry.

Changing Minds through Reform Curriculum-Creating Problem Solvers

The way mathematics is taught in this classroom allows for the students to develop their thinking, and be persistent problem solvers.

There are many activities in our classroom geared toward the goal of helping children think about math, and not just "do what I tell you to do". By asking the

children to decide how they want to solve problems, we allow them the chance to think about what possible ways there could be to solve a particular problem. We believe our children's thinking has changed because the strategies many are using now are not the same ones they were using at the beginning of the year. Since we avoid demanding that the students solve problems "our way", our conclusion is this shift is due to their development as thinkers and problem solvers.

Second graders are supposed to become proficient with these mathematical topics: adding and subtracting basic facts to 18, adding two digit numbers with regrouping, subtracting two digit numbers with regrouping using manipulatives, and adding and subtracting three and four digit numbers without regrouping.

In our classroom we teach strategies to help kids learn their basic number facts to 20. Chapter 8, Helping Children Master the Basic Facts, in *Elementary and Middle School Mathematics* lists four big ideas dealing with the basic facts.

1. Number relationships can be used to help remember basic facts.
2. For subtraction facts, the concept "think addition" is the most important idea.
3. There are patterns and relationships in basic facts. You can figure out new or unknown facts from the ones you already know.
4. All the facts can be learned with the help of efficient strategies.

(Van de Walle, 1998, p. 140)

We use strategies as ways to learn the basic facts, rather than just memorize them. For example, facts that "add to 10" is our first strategy. Most kids worked on this in first grade, yet we find they need much more practice with 6 + 4, 8 + 2, etc. We play games and do problems based upon these "ten facts". Once the kids understand what parts add up to 10, they are able to use this to solve other problems. A child may say, "Since 8 + 2 is 10, then 8 + 3 is 11." In the book *Developing Number Concepts: Addition and Subtraction*, there are many activities which I believe could help kids practice our strategies with manipulatives. Many children still count to solve these problems, especially early in the school year. Richardson talks about counting as the first strategy children use. In our classroom we allow kids to choose how they wish to solve problems. *Investigations* encourages kids to decide how they want to solve the problems. For many, this means "counting all" at first, usually with some sort of manipulative. In *Children's mathematics: Cognitively guided instruction*, the development of strategies is described like this:

> Most children pass through three levels in acquiring addition and subtraction problem-solving skills. Initially they solve problems exclusively by Direct Modeling. Over time, Direct Modeling strategies are replaced by the use of Counting strategies, and finally most children come to rely on number facts. The transition from Direct

> Modeling to using Counting strategies does not take place all at once, and for a time children may use both... Similarly, children learn a few number facts quite early... and the use of recall and Derived Facts evolves over an extended period of time. (Carpenter et al, 1999, p. 26)

The next strategy we see, sometimes very early in the year, is what we call "count on" or "count back". Some arrive at this strategy on their own; some pick it up after hearing others share their strategies. For the most part kids start using the strategy with manipulatives, including fingers. For example, a child may start with 12 and count on 5 like this: (12) 13, 14, 15, 16, 17. The answer is 17. We provide many opportunities to solve a variety of problems, most of which are with numbers below 20, at least near the beginning of the year. As time goes on, we increase the magnitude of the numbers. When talking with other teachers about the work we do in our classroom, many are concerned about the fact that our kids really focus on problems whose numbers are under 100. Yet the information in both *Investigations* and Developing *Number Concepts* states that kids need to work with lower numbers first, numbers that may range from 10 to 30 or 40. Once kids develop the understanding of place value and have figured out strategies for solving problems, they can then apply these to numbers up to and over 100. As Richardson notes:

> Through their work with smaller numbers, children build the

> strategies that they can then apply to their work with larger numbers. (Book 2, 1999, p. 101)

There is also the issue of moving them beyond counting. Some children will stay with the counting strategy and not use others unless encouraged.

> The problem is not that children count but rather that many of them do not move beyond counting. Our role is to help children notice, understand, and internalize number relationships. (Richardson-book 2, 1999, p. 101)

We do this in a couple ways in our classroom. First, by having the students share their strategies, they are exposed to other ways of thinking. They are encouraged to try them if they wish. Second, we ask them to solve some of their problems in more than one way, especially nearer to the beginning of the year. The third way we help students is by continually reminding them that there are many ways to solve problems, and no one way is "right", which would make all other ways "wrong". This allows the students some freedom to decide how they wish to solve the problems. These ideas are from the *Investigations* series, but they are also present in Richardson's *Addition & Subtraction* book.

Using CGI reinforces the ideas presented in Investigations and takes them a

step further, which can help a teacher become more thoughtful about their mathematics teaching.

One area we focus on extensively is helping the children <u>think</u>. We want them to think about what is happening in the problem, so they understand what it is asking them to do. We want them to think about how they would like to solve the problem. (There is almost always more than one strategy that will work!) We want them to think about their process, and see if it made sense for the problem. We also want them to think about their answer in terms of whether it makes sense for the problem.

> If from and early age, children are taught to approach problem solving as a way of making sense out of problem situations, they may come to believe that learning and doing mathematics involves the solution of problems in ways that are always meaningful. (Carpenter et al, 1999, p. 57*)*

When we first started our inquiry, we were unsure how problem types in CGI would compare to those in Investigations. We also thought that some of the problem types in CGI would be harder for the kids to solve than others. What we found was that through *Investigations,* the students had completed similar problem types for some of the CGI problems. We were surprised at how well the students did with many of the problems. Even the ones we thought would be very difficult,

almost half of the class solved correctly. (See Appendix B.3) We did notice the students tended to do better on problems we might consider to be adding, the "join" problems. If we averaged the number correct for the three "join" problems, and compared that with the average for the three "separate" problems, they scored a 76% for "join" problems, compared to a 55% for "separate". Most second graders do have more successes with addition. "Subtraction facts prove to be more difficult than addition." (Van de Walle, 1998, p. 149)

Teaching mathematics using a reform curriculum provides students the opportunity to challenge their thinking and build confidence.

The *Investigations* Series we use in our classroom allows students to choose the method for solving problems that works best for them. Our students realize that there is not one "correct" way to solve a problem. They can experiment with a plethora of different strategies to solve problems and use the one that works best for them. One of our students who just moved here in February had this to say, "I think the math we do here is better because you get to solve it the way you want and it is easier." (See Appendix C) The students really benefit from being able to choose the method they want to use to solve problems because not all of our students are on the same level. By allowing them to pick how they want to solve

the problem, they can all solve the same problem even if some students are using more complicated techniques, essentially building their confidence.

Students need to be given the chance to challenge themselves in order to further their learning and understanding. Everyone received problems with the same numbers in them, even students that some would consider to be "weaker". We struggle between giving numbers that are low enough for the kids who choose counting, and high enough to be challenging. In *Investigations*, a teacher note in "Putting Together and Taking Apart" helps guide the teacher to think about the numbers.

> Start with numbers suggested for each problem and then adjust the numbers after students have solved the basic problems. In this way both you and students have some immediate information about the level of difficulty of the problems. Sometimes you can choose the new numbers. At other times, let students choose appropriate numbers for themselves from some you suggest. (Economopoulos et al, 1998, p. 12)

If teachers give needier students less challenging numbers because they are afraid "they will get the wrong answer," then those students are not able to blossom. Van de Walle puts it this way:

> Not all children will develop the same problem-solving abilities, but all have abilities and can contribute. This must be something you truly believe because it is difficult to fake. (Van de Walle, 1998, p. 59)

Linda and I agreed that we would give all of the students the problems as they were written and if someone was having extreme difficulty we would change the numbers. We did not end up changing any numbers for students, although a few students were not sure how to solve some of the problems. For the most part, these students were able to work on the problems once an adult helped them figure out a strategy.

No two students learn the same way and that is something a teacher must always remember. The wonderful thing about having the students do these problems is *they* get to choose how they want to solve the problem.

Students benefit from sharing strategies with their classmates.

The *Investigations* Series provides time at the end of many sessions for students to reconvene and discuss the strategies they used to solve the problems. The students are able to learn different strategies to solve problems by listening to their classmates. We try to make sure different methods for solving problems are represented so the students can see a variety of ways to solve the problems.

We also try to give the students opportunities to switch papers with another student and see if they can solve the problem differently. One student said, "This year she got different ideas and strategies to solve problems from swapping papers with other students." (See Appendix C) Sharing strategies has worked particularly well in our classroom because it enables the students to visualize other ways they may like to try when solving problems. Then the next time they solve problems they may opt to use a technique they learned from one of their classmates. *Van de Walle* reminds us to make sure we take the time to listen to various students' ideas.

> When students volunteer ideas, listen carefully and actively to each idea, and give credit for the thinking and the risk that children take by venturing to speak out. Be careful to focus praise on the risk or effort and not the products of that effort… Avoid ending a discussion with the first correct answer… If ten hands go up, ten children may have been doing good thinking. (Van de Walle, 1998, p. 59)

Changing Minds and Shifting Feelings-The Affective Side

The students in our class have a positive attitude about mathematics and enjoy the way we do math in our classroom.

The individual student interviews allowed the students to explain their

feelings about mathematics in greater detail than the student surveys. Based on the interviews every student said at least one positive statement about mathematics. For example, one student said "I didn't like math last year and now it's my favorite subject." (See Appendix C) This student has changed his opinion of math during this year from one extreme to another. That kind of transformation is not common, especially in such a short amount of time. Another student had this to say, "I like solving problems and figuring out problems using different ways." (See Appendix C) She enjoys using different manipulatives as well as mental processes to solve problems.

Every student in our class feels they are good at some component of mathematics.

During the individual interviews every student expressed their belief that they are good at doing at least one aspect of mathematics. For example, one student said "I think I'm good at math because I can do addition problems very quickly." (See Appendix C) Her ability to solve addition problems quickly stems from allowing her to solve them on her own terms. Another student said, "I think I'm good at addition and subtraction." (See Appendix C) It is so important that the students have confidence in

themselves as mathematical thinkers/learners in order for them to excel in the area and be successful. The fact that every student could identify at least one part of math they are good at says a lot.

Some of the students in our class experienced a different type of mathematics in their classroom last year, but all of them feel they are better at math this year.

Most of the students said they learned math differently last year than they are learning math this year. Many of them said they did not know how to do number strings last year, although some said they did similar math activities in their class. However, the truly fascinating part is that every student feels they are better at math this year. One child said, "I think I've gotten better at math this year than last year because of the strategies we use." Another student thinks the strategies we use this year have helped her become better at math "because last year I didn't understand the tens as well as I do now." (See Appendix C) We must be doing something right if second graders are noticing a difference in their mathematical abilities because of the strategies we use.

Changing Minds-Dani's Discoveries

Teaching mathematics using a reform curriculum has provided me the opportunity to build confidence in my mathematical thinking and teaching.

Now that I have been "forced" to teach mathematics in my classroom, I have been trying to be open-minded. It can be difficult to put aside my prior experience, but it is worth it to ensure that my students have better experiences with mathematics than I did. I am fortunate that my mentor teacher is an advocate of reform mathematics. I have been using *Investigations* to plan my lessons and they have been really helpful. I am glad that I am learning to teach mathematics using thought provoking techniques. This method of teaching mathematics will be tremendously advantageous when I have my own classroom. *Investigations* provide opportunities to challenge students' thinking while allowing for multiple methods to solve the problems. Therefore, students who are at a lower mathematical level can still use the techniques they are comfortable with, while being introduced to more complex forms for solving problems.

Learning how to teach mathematics using *Investigations* has been extremely beneficial. I am amazed to hear the way my students are solving problems and to

watch them grow in their mathematical understanding. Sometimes the students do not even realize what types of mathematics they are learning. I believe that *Investigations* offer students the opportunity to feel confident, regardless of their ability level. Mathematics is the subject I am the least comfortable teaching my students because I feel it is my weakest. However, my confidence and ability to "think outside the box" when it comes to mathematics is improving because of the way I am learning to teach. I wish that when I was growing up I learned mathematics this way. If I had, I think my confidence would be higher and I would have enjoyed it a lot more. (See Appendix D)

Teaching mathematics using a reform curriculum has made math more enjoyable for me.

Using reform mathematics has in a sense restored my faith in myself as a math learner. As the students learn to solve the problems using different techniques, I too am learning new ways to solve problems. I never knew how to do a number string until this year. I am finding the *Investigations* Series to be extremely beneficial not only for the students, but for myself. Every day I am learning a new mathematics technique either from Linda or my students. I am also

coming to new realizations on my own as I continue to teach my students. (See Appendix D)

Conclusions and Future Directions

Rodger

My position as PDA prohibited my daily participation in the inquiry. The loss is mine. I made many observations that will spur me to continue to look at children in this new way in my classroom next year.

Classroom teachers are busy, but I must find some way to find the time to observe children as they solve problems. Most importantly I must learn to remain quiet for long periods of time as they work. My natural urge to ask questions or "help" them must be suppressed in order to see the richness of their thinking. That does not mean that I have no role or cannot help them, but the times when I stop the child to make them think "my way" will be reduced.

Alternate algorithms also seem to be a worthwhile pursuit next year. I wonder how useful they would be in third grade. Linda has her students use "number strings" as a pathway to the more traditional algorithms. I even witnessed, on occasion, that she allows students to create their own ways to work

with numbers. It is only meant to be a temporary "scaffolding" for the students. When Linda feels they are ready for a higher level of abstraction she then introduces them to the "short form" algorithms that our American culture values.

I will also endeavor to use problem-solving daily in my classroom. Problem solving was treated as a special topic in my room. It was reserved for Fridays and we emphasized "strategies" such as “Making a Chart” or “Thinking Backward.” I am thinking of using problem solving as the keystone to my program. I wonder if this emphasis on problem solving will help my students take the third grade PSSA Math tests next year.

My math program will include less "ability grouping" next year. I know that Linda has used small groups in her room. I also know that she is aware of the different abilities of her students, yet the activities each day were broad enough to include all of the children in the room. Dani observed that the students all seemed to feel confident in their abilities and all seem to enjoy math. I believe those feelings are in response to the community that is built when the children are not ability grouped and are allowed to work at their level and their pace on the problems.

Finally, I wonder if there is a way for teachers to meet on a regular basis to discuss these new ways of looking at children's mathematical thinking.

Investigations in Number Data and Space, a reform mathematics program, is going to be piloted in kindergarten through second grade next year. Teachers will need time to discuss the pilot and their thinking about teaching math in a more child-centered way. As a PDA this year I received training in a program entitled Critical Friends Groups (CFG). A brief explanation follows:

> CFGs generally consist of 6-10 educators who meet regularly for a sustained and focused period of time to work and learn together, and who observe each others practice, examine each others work, and give feedback to each other on a regular basis. In addition administrators and teacher-leaders in the school and district work as facilitative leaders, developing good habits in colleagues by routine use of National School Reform Faculty (NSRF) practices and tools in staff meetings, cabinet meetings, planning sessions, grade level meetings, department meetings, and other kinds of professional development sessions.

I wonder if my training in CFG will help facilitate the adoption of the Investigations Math program next year.

Linda

I would be the first to admit to my enthusiasm concerning teaching through a program like *Investigations*. Sometimes I wonder if my enthusiasm blocks me from considering in depth what I've chosen to do with the children. The reading I've done so far has reassured me that my teaching is "on the right track". It has also reinforced my desire to continue to teach for understanding, and to continue learning about reform mathematics. I am going to keep reading through *CGI* as well, because it helps describe the order in which children typically develop strategies for solving problems. One sub question I did not get to complete was looking at this continuum to see if it can help me develop problems that would guide kids to the next step when they are ready. It would have been helpful to list the types of strategies the children used over time and see if they have changed. I would like to spend some time next year keeping better records on which strategies the students choose to use.

Another sub question focuses on whether the activities in *Investigations* are carried through the various modules. I am definitely going to get a chance to explore this next year, since our district is piloting the use of *Investigations* in kindergarten, first and second grade classrooms. I am anxious to see if the current math curriculum will be revised, too, so that I will be able to use each

Investigations module without feeling the pressure to include everything in our current curriculum. *Investigations*, and other reform curriculum certainly are not perfect; each most likely has areas that are stronger, and some that are weaker. Yet using a curriculum such as *Investigations* gets us much closer to teaching for understanding than traditional programs. It will be interesting to see how other teachers feel about it after they have had a chance to use it.

Dani

My position as a PDS intern extremely enhanced my inquiry. I was fortunate to have the opportunity to sit with small groups of children as well as individual children and watch them solve problems. I was able to question their techniques to learn more about how they chose to solve problems and the thinking that was involved in their decisions. The students in my class have come a long way in many areas of mathematics since the beginning of the year.

As I continue my career as an educator I look forward to implementing a reform mathematics curriculum in my own classroom. I intend to use problem-solving daily in my classroom. A huge smile is brought to my face every time I watch students become ecstatic because they figured out how to solve a

challenging problem. Watching students work through problems that they were stuck on has been extremely rewarding.

Challenging myself as a math teacher and learner is something I desire to continue. I must say I never thought I would enjoy mathematics. However, the reform mathematics curriculum we use in my classroom has really opened my eyes to the wonderful things students are capable of if provided the necessary opportunities. My students have been reassured throughout the year that they may solve problems using whatever method they find most comfortable. By allowing students this choice and assuring them that their thinking is more important than the correct answer, we are taking away the pressure that often comes with mathematics. If I had learned mathematics this way, it would have been much more pleasurable.

As I reviewed the student surveys I was surprised by some of the students' answers, but expected some to have answered the way they did. I started wondering if the wording of the questions affected the students' answers. The interviews got me thinking, are the students' attitudes toward mathematics applicable to other subject areas? It would be interesting to see if I asked the students the same questions pertaining to different subjects, would they give the same responses? I will definitely continue questioning where to draw the line

between "telling" the students the answer and letting them figure it out for themselves. It is important to let students experiment and draw conclusions on their own, but there comes a point where you do not want them to struggle any more than necessary.

Future Changes

Inquiry is an ongoing, long-term process. We have certainly learned from our inquiry, yet in some ways we feel we have only just started. We do feel "all the minds are changing", including ours, and hopefully will continue to change as we keep looking at children's mathematical thinking. Each of us looks forward to continuing our inquiries, both this year and in future years.

Appendix A.1

Name_____Date_____

Directions: Read each of the following sentences carefully. Look at the faces. Circle the face that best matches how you feel about that sentence.

D. I like Math because I can figure things out.

2. I keep on working when a problem is very hard.

3. The only thing I worry about in Math is getting the right answer.

4. Math is very useful even outside of school.

5. I like to explain how I got my answers in Math.




Put a √ on the line below where you think you would belong.

I am not good |_____|_____|_____|_____|_____|_____|_____|_____| I am good at Math.

What parts of math do you think you do well?

Appendix A.2

Dear Room 21 Families,

As you are aware, we've been using many activities from our resource called "Investigations in Number, Data and Space" as part of our math program. We've been amazed at how far the students have come in their willingness and ability to explain their answers. We'd like to get your opinion of your child's math, and how he/she feels about it. (We will be asking the students' opinions as well in school.) Please take a few moments to answer the questions below. We'd like to have these by Friday, March 19th, if possible, so we can start compiling them.

Let us know if something is not clear!

Thank you!
Linda Margusity & Dani Sturtz

1. How do you think your child feels about math this year?

2. How often does your child share what we've done in class with you? Is he/she able to explain it clearly?

3. Does the math our child is doing currently remind you of the math you did in elementary school? Please explain.

4. Do you have any other comments about your child's mathematics this year?

Appendix A.3

Name_____ Date:

Read each problem carefully. Underneath each problem is a blank space. Show your work in the blank space. You may use counters, 100's charts or other things to help you solve the problems. Good Luck. ☺

A. Ben has 9 marbles. His father gave him 17more marbles. How many marbles does Ben have now?

B. Dani had 23 coins in her collection. Her mother gave her some more coins. Now Dani has 51 coins. How many coins did her mother give her?

C. Yesterday Sam's dad gave him 17 baseball cards. Now Sam has 42 cards. How many cards did Sam have before his dad gave him some cards?

D. Linda had 56 gummy bears. She ate 19 of them. How many gummy bears does she have now?

E. Jacques had 43 Matchbox cars in his toy box. He gave some of them to his cousin. Now he has 28 cars. How many cars did he give to his cousin?

F. Julia had some cookies. She shared 14 of them with her friends. Now she has 37 cookies. How many cookies did she have before she shared some with her friends?

G. There were 12 soccer trophies in the school trophy case. There were 39 blue ribbons in the case. How many awards are there in the school trophy case?

H. 53 frogs were sitting on rocks around the pond. 17 of the frogs had spots. The rest of the frogs did not have spots. How many frogs did not have spots?

I. 6 friends were eating lunch at the park. Each friend had 7 things to eat for lunch. How many things were there to eat in all?

J. Grace had 42 brownies that she helped bake for the sale. She wanted to put them into packs. Each pack would have 3 brownies in it. How many packs could she make?

K. Franco had 45 insects in his collection. He was tired of collecting bugs and wanted to give them away to three friends. How many bugs would each friend get if Franco passed out the same number of bugs to each friend?

Appendix B.1
Student Survey Results
(scale: 4= most positive comments to 1= least positive comments)

NAME	QUESTION 1	QUESTION 2	QUESTION 3	QUESTION 4	QUESTION 5	**TOTAL/CHILD**
Reyna	4	3	1	4	4	**3.2**
Daniel	3	2	4	4	2	**3**
Emma	3	3	4	4	3	**3.4**
Rachel	3	4	3	4	4	**3.6**
Taylor	4	4	3	4	3	**3.6**
Marissa C.	3	2	2	2	3	**2.4**
Angelika	3	2	3	2	3	**2.6**
Kodie	3	4	4	4	4	**3.8**
Luke	4	3	4	2	1	**2.8**
Brittany	4	4	3	4	4	**3.8**
Nicholas	3	1	1	4	1	**2**
Sarah	4	4	3	3	2	**3.2**
Darya	4	4	2	2	4	**3.2**
Betsy	4	3	3	4	1	**2.75**
Marissa	4	4	4	3	4	**3.8**
Sean	3	2	1	4	4	**2.8**
Brynn	4	4	4	4	4	**4**
Samantha	2	4	4	2	3	**3**
Natalia	4	2	2	2	4	**2.8**
Jake	3	4	2	4	3	**3.2**
Amber	4	3	2	3	3	**3**
Total /column	**3.48**	**3.14**	**2.81**	**3.29**	**3.05**	**Average = 3.15**

Appendix B.2
Parent Survey Results
(scale: 4= most positive comments to 1= least positive comments)

NAME	QUESTION 1	QUESTION 2	QUESTION 3	w/ # 3	QUESTION 4	**TOTAL/CHILD**	COMMENT
R.							none
D.							none
E.	4	4	2.5	yes	4	**3.63**	
R.	4	4	4	no	4	**4.00**	
T.	4	3	4	no	2.5	**3.38**	# 4 comment
M.	4	3	1	no	1	**2.25**	
A.	4						none for 2-4
K.							none
L.	4	2	4	no	3	**3.25**	
B.	4	4	3	no	4	**3.75**	
N.	4	2	4	no	4	**3.50**	
S.	4	4	4	no	4	**4.00**	
D.							none
B.	4	3	4	no	4	**3.75**	
M.	4	4	4	no	4	**4.00**	
S.	4	2	2.5	no	4	**3.13**	
B.	4	4	4	no	4	**4.00**	
S.	4	2	1	yes		**2.33**	#4: No
N.							none
J.	4	3	4	no	4	**3.75**	
A.							none
Total /column	**4.00**	**3.14**	**3.29**		**3.32**	**3.48**	

Appendix B.3
Student Problem Results

CGI Problem Type	Number correct	Percent	In Investigations?	Which book?
JRU: Join (Result Unknown)	18/21	86	Combining (Unknown Outcome)	(C,C, & C and PTTA)
JCU: Join (Change Unknown)	14/21	66	Combining (Unknown Change)	(PTTA)
JSU: Join (Start Unknown)	16/21	76	(not mentioned as a type, but problems could be developed)	
SRU: Separate (Result Unknown)	10/21	48	Separating (Unknown Outcome)	(C,C, & C and PTTA)
SCU: Separate (Change Unknown)	12/21	57	Separating (Unknown change)	(PTTA)
SSU: Separate (Start Unknown)	12/20	60	(not mentioned as a type, but problems could be developed)	
PPW WU: Part-Part Whole (Whole Unknown)	15/21	71	Considered a combining problem	(C,C, & C and PTTA)
PPW PU: Part-Part Whole (Part Unknown)	9/18	50	Considered a separating problem	(C,C, & C and PTTA)
M: Multiplication	9/20	45	no (but recurring activity "How Many Pockets" is)	
DP: Division (Partitive)	12/20	60	no	
DQ: Division (Quotative)	10/19	53	no	

Join is what we typically would call addition

Separate is what we typically would call subtraction

Combine is what we typically would call addition

Separating is what we typically would call subtraction

(C, C, & C is Coins, Coupons & Combinations)

(PTTA is Putting Together and Taking Apart)

Appendix C

(Copies of selected student interviews, unable to be attached to archived version. See Appendix D for the next section.)

Appendix D

Dani Sturtz
2/17/04

Mathematics is a subject I have struggled with since elementary school. I have always felt that I am terrible at mathematics and therefore did not enjoy the subject. During math I prayed my teacher would not choose me to share my answer. I feared that it would be wrong and that my classmates would laugh at me. My lack of confidence in myself as a mathematics learner has definitely contributed to my negative feelings about math.

The way in which I was taught mathematics is also a huge factor in my "hatred" of the subject. My teachers were very traditional and they taught "just to teach" as opposed to teaching for understanding. Instead of focusing on how I attempted to solve the problems and the methods I used, my teachers were concerned with my answers. I was expected to solve problems their way and was never given the opportunity to explore other options. Following a formula to solve a problem was a key component throughout my mathematical career. Many times if you could not use the formula correctly, you could not solve the problem.

Now that I have been "forced" to teach mathematics in my classroom, I have been trying to be open-minded. It can be difficult to put aside my prior experience, but it is worth it to ensure that my students have better experiences

with mathematics than I did. I am fortunate that my mentor teacher is an advocate of reform mathematics. I have been using "Investigations" to plan my lessons and they have been really helpful. I am glad that I am learning to teach mathematics using thought provoking techniques. This method of teaching mathematics will be tremendously advantageous when I have my own classroom. "Investigations" provide opportunities to challenge students thinking while allowing for multiple methods to solve the problems. Therefore, students who are at a lower mathematical level can still use the techniques they are comfortable with, but are also being introduced to more complex forms for solving problems.

Learning how to teach mathematics using "Investigations," has been extremely beneficial. I am amazed to hear the way my students are solving problems and to watch them grow in their mathematical understanding. Sometimes the students do not even realize what types of mathematics they are learning. I believe that "Investigations" offer students the opportunity to feel confident, regardless of their ability level. Mathematics is the subject I am the least comfortable teaching my students because I feel it is my weakest. However, my confidence and ability to "think outside the box" when it comes to mathematics is improving because of the way I am learning to teach. I wish that when I was

growing up I learned mathematics this way. If I had, I think my confidence would be higher and I would have enjoyed it a lot more.

Dani Sturtz
3/19/04

This week during math we have been giving our students problems to assess their understanding and procedure for completing the problems. Before we gave the students the problems, my mentor teacher and I went over the paper and discussed if the numbers were appropriate for the students. When I first looked at the paper I felt that some of the numbers would be too high for certain students. However, then I realized that it is okay if they were not able to do the problem. The point is for them to be given the opportunity to try.

Students need to be given the chance to challenge themselves in order to further their learning and understanding. By giving the weaker students the greater numbers they are receiving the same option as everyone else. No one said they have to come up with the correct answer. Maybe they won't, but maybe they will. If teachers do not give the weaker students the higher number because they

are afraid "they will get the wrong answer," then they are not allowing students to blossom. My teacher and I agreed that we would give all of the students the problems as they were written and if someone was having extreme difficulty we would change the numbers.

Thus far, it does not appear that any student has needed to have the numbers changed. Some students are having an easier time than others, but that is to be expected. No two students learn the same way and that is something a teacher must always remember. The wonderful thing about having the students do these problems is <u>they</u> get to choose how they want to solve the problem. Therefore, the student can use the method that is most comfortable.

There are many different ways I am noticing my students are choosing to solve the problems. Some students are counting through use of tallies, while others are counting on a hundreds chart or counting items. Other students are using number strings. The method the student uses to solve the problem helps my mentor teacher and I see the students' level. By reviewing these problems we are able to see what the students understand and what they still need to practice. We are able to determine our next step for teaching by the students' responses to the problems.

One thing I have noticed particularly while watching the students work on the problems is the difficulty they are having explaining their answers. The students are not permitted to just write they answer, but they must justify their answer by showing work. Some applications such as numbers strings do not need a written response. However, if the students used a hundreds chart they must explain where they started and ended and how they came up with the answer. It can be extremely difficult for young students to process and write their thinking. That is something we have been working on with our students.

When the students actually do explain themselves, it is amazing to hear what they write. I am learning that my students are capable of so much more than they know. Their responses have blown me away at times. When I was their age I do not know if I would have been able to explain my answer to the extent that my students do. As I said it can be very difficult for some students to explain their thoughts so how can I make it easier for them without telling them what to write? Also, sometimes students get stuck and say they do not know how to solve the problem. I try to give them ideas without telling them how to do the problem, but how can I make sure I am not saying too much?

Appendix E
Resources

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). Children's mathematics: Cognitively guided instruction. Portsmouth, NH: Heinemann.

Dana, N. & Yendon-Silva, D. (2003). The reflective educator's guide to classroom research.

Interviews with second grade students, Panorama School, April, 2004.

Investigations in number, data, and space, Suggested grade 2 sequence: Economopoulos, K., Akers, J., Clements, D., Goodrow, A., Moffet, J., & Sarama, J. (1998). Mathematical thinking at grade 2. White Plains, NY: Dale Seymour Publications.

Economopoulos, K., & Russell, S. (1997). Coins, coupons and combinations. Palo Alto, CA: Dale Seymour Publications.

Russell, S., Corwin, R, & Economopoulos, K. (1998). Does it walk, crawl, or swim? Menlo Park, CA: Dale Seymour Publications.

Akers, J., Battista, M., Goodrow, A., Clements, D., & Sarama, J. (1998) Shapes, halves and symmetry. Menlo Park, CA: Dale Seymour Publications.

Economopoulos, K., & Russell, S. (1998). Putting together and taking apart. White Plains, NY: Dale Seymour Publications.

Goodrow, A., Clements, D., Battista, M., Sarama, J., & Akers, J. (1998) How long? How far?. Menlo Park, CA: Dale Seymour Publications.

Economopoulos, K., Wright, T. (1997). How many pockets? How many teeth? Palo Alto, CA: Dale Seymour Publications.

Wright, T., Nemirovsky, R., & Tierney, C. (1998). Timelines and rhythm patterns. Menlo Park, CA: Dale Seymour Publications.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Stud Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: national Academy Press. 353-54.

Richardson, Kathy (1984) Developing number concepts using unifix cubes. Menlo Park, CA: Addison-Wesley Publishing Company.

Richardson, Kathy (1999) Developing number concepts, book one: Counting, comparing, and pattern. Parsippany, NJ: Dale Seymour Publications.

Richardson, Kathy (1999) Developing number concepts, book two: Addition and subtraction. Parsippany, NJ: Dale Seymour Publications.

Richardson, Kathy (1999) Developing number concepts, book three: Place value, multiplication, and division. Parsippany, NJ: Dale Seymour Publications.

Survey by parents of second grade students, Panorama School, March 2004.

Survey by second grade students, Panorama School, March 2004

Van de Walle. J. (1998) Elementary and middle school mathematics: Teaching developmentally (third edition). New York, NY: Addison Wesley Longman, Inc.